Scattering a quantum particle on the potential step: Characteristic times for its subprocesses – transmission and reflection

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Abstract. A new model of scattering a quantum particle on the potential step, which allows tracing the dynamics of its subprocesses – transmission and reflection – at all stages of scattering, is developed. Within this model, on the basis of the flow and group velocity concepts, we study the temporal aspects of these subprocesses. In particular, for both subprocesses the dwell and asymptotic group times are defined. Unlike the conventional model of this process ours predicts the existence of a transient zone, immediately after the step, where the average velocity of particles varies monotonously between its asymptotic values. Due to this zone the delay times calculated for transmitted and reflected particles are nonzero for the potential step, while the conventional model predicts zero values. By our model, only the flow velocity concept gives the true velocity of transmitted particles at all stages of scattering.

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1. Introduction

As we are aware, in the tunneling time literature (TTL) there are two timekeeping procedures [1, 2] that deal directly with the temporal aspects of scattering a quantum particle on the potential step. Being different in some respects, both these procedures lead to the same result – the average velocity of a particle to travel above the step is, like the potential itself, a piecewise constant function of a spatial variable. And, at first sight, this result should be considered as well established because these approaches are clock-based, i.e., they not only introduce characteristic times for this potential but also suggest the way of their measuring. However, this result is unacceptable from the physical point of view – the average velocity of particles with nonzero rest mass cannot be changed instantaneously at the step. This tells us once more that not any theoretical model that provides the scheme of its experimental test is indeed true – the model must be also internally consistent as well as consistent with the well-established fundamental physical laws.

At the same time, of widespread in the TTL is the thought (see, e.g., the original papers [3, 4] and reviews [5, 6, 7] on this problem) that the TTP cannot be consistently resolved within quantum mechanics (QM). Among reasons to make its consistent resolution impossible the following two are usually pointed out: the peculiar role of time in quantum mechanics, and impossibility to trace the transmission and reflection subprocesses at all stages of scattering because of the irremovable interference between them.

However, there is a reason to believe that only the latter is significant. Indeed, one has to take into account that the TTP appears not only in QM but also in classical electrodynamics (CED) as well as in all those branches of classical physics, that deal with the propagation and scattering of waves in layered structures and media. It is obvious that searching for the time operator for solving the TTP is inappropriate in these cases. Thus, the TTP should be considered as a problem of *wave* mechanics, rather than *quantum* mechanics only – the way of its resolution must be common for waves of any nature.

As is stressed in [8], a consistent definition of the tunneling (transit) time for a quantum particle scattering on a one-dimensional potential barrier is impossible within the conventional quantum-mechanical model (CQM) of this process, because it does not allow tracing the dynamics of the transmitted part of the incident wave packet at all stages of scattering. To fill this gap in the CQM, all the existing approaches to the TTP resort to some 'self-evident' (unproven) assumptions about its dynamics at the initial stage as well as at the very stage of scattering, which are incorrect on closer inspection. In detail, all such assumptions are analyzed in the paper [8].

Of most importance is that it is assumed as self-evident that the time evolution of the transmission subprocess, like the whole process to involve both transmission and reflection, is unitary at all stages of scattering. However, this is fundamentally wrong. Indeed, if this would be true, then even in the non-resonant case the transmission

subprocess could take place separately from reflection. What is impossible in principle.

In the papers [9, 10, 11, 12] we present a novel approach to the tunneling time problem, where scattering a quantum particle on a symmetric one-dimensional potential barrier is treated as a complex process consisting of two inseparable coherently evolving subprocesses - transmission and reflection. This approach allows tracing the transmission (and reflection) dynamics at all stages of scattering, thereby proving that the widespread assumption on the indistinguishability of these subprocesses is mistaken. As is shown, in the stationary case the total wave function to describe this process can be uniquely decomposed into the sum of two 'subprocess wave functions' (SWFs) to separately describe transmission and reflection in all spatial regions. Their main peculiarity is that either of them consists of one incoming wave and one outgoing wave, joined 'causally' at the midpoint of the barrier region. The word 'causally' means that each SWF is continuous at this point together with the corresponding probability current density (rather than with its first spatial derivative).

As was shown in [8], this weakened continuity condition guarantees the unitarity of the quantum dynamics of each subprocess in the stationary case. In the time-dependent case the situation becomes more complicated. Namely, the time-dependent reflection dynamics is again unitary. However, the transmission dynamics is unitary only in the 'asymptotical' sense – the wave function to describe the transmission subprocess at the initial and final stages of scattering has the same constant norm; however, at the very stage of scattering, its norm varies due to the joining point to serve as a sink or source of particles for the transmission subprocess. In the case of symmetric barriers this role is played by midpoints of their barrier regions.

Due to this feature of the transmission subprocess the group-velocity concept to deal with tracing the center of 'mass' (CM) of wave packets does not allow revealing the true *tunneling* velocity and time that describe transmitted *particles* in the barrier region. By our approach, only the concept of the (probability or energy) flow velocity and associated concept of the dwell time can be used for this purpose.

Besides, by our version of the Larmor-clock procedure, the average spin of electrons to pass through the symmetric barrier (with the infinitesimal magnetic field switched on in this region) experiences in the barrier region not only a smooth Larmor precession but also a sudden 'flip' at the midpoint of this region (of course, for the infinitesimal interval dt of the time evolution this 'flip' is infinitesimal too). This effect makes it impossible a direct measurement of the transmission dwell time by means of this procedure. However, with making use of all relationships obtained in our version of the Larmor-clock procedure, this characteristic time can be measured indirectly.

In this paper, we first extend our approach onto the most simple *asymmetric* potential barrier – the potential step. In studying the temporal aspects of this potential, we will use both the flow and group concepts of velocity.

2. Wave functions for transmission and reflection

So, let a particle with the energy $E = \hbar k^2/2m$ impinges from the left on the potential step of depth V_0, placed at the point $x = a$ ($a > 0$). The total wave function $\Psi_{tot}(x,k)$ to describe this stationary scattering process for $E > V_0$ is

$$\Psi_{tot}(x,k) = \begin{cases} e^{ikx} + Be^{-ikx} & x < a \\ Ae^{i\kappa x} & x > a \end{cases} \tag{1}$$

$$A = \frac{2k}{k+\kappa}e^{i(k-\kappa)a}, \quad B = \frac{k-\kappa}{k+\kappa}e^{2ika};$$

$$\kappa = k\sqrt{1-\beta(\kappa_0/k)^2},\ \kappa_0 = \sqrt{2m|V_0|}/\hbar,\ \beta = sgn(V_0).$$

In this case the transmission and reflection coefficients are

$$T = \frac{4k\kappa}{(k+\kappa)^2}, \qquad R = \left(\frac{k-\kappa}{k+\kappa}\right)^2$$

In line with the approach [9], the wave functions $\psi_{tr}(x,k)$ and $\psi_{ref}(x,k)$ for the transmission and reflection subprocesses must obey the following requirements:

(a) $\psi_{tr}(x,k) + \psi_{ref}(x,k) = \Psi_{tot}(x,k)$;

(b) either of these two SWFs has one incoming and one outgoing waves, joined at some point $x_c(k)$;

(c) either of them is continuous at this point together with the probability current density.

As it turns out, in the case of the potential step these requirements lead to the unique pair of functions $\psi_{tr}(x,k)$ and $\psi_{ref}(x,k)$ whose incoming and outgoing waves are joined at the point $x_c(k) = a + \kappa^{-1}\arctan(\kappa/k)$. This point is such that the phase path $\kappa(x_c - a)$ tends to $\pi/4$ when $k \to \infty$, for both signs of V_0; in this case $x_c \to a + 0$. At the same time, $\kappa(x_c - a)$ tends to zero when $V_0 > 0$ and $k \to \kappa_0 + 0$, but $\kappa(x_c - a)$ tends to $\pi/2$ when $V_0 < 0$ and $k \to 0$; in these two cases $(x_c - a) \to \infty$.

Thus, by our model a nonzero probability to find reflected particles to the right of the step takes place only in the transient zone $[a, x_c(k)]$ – no one reflected particle with a given energy $E(k)$ crosses the point $x_c(k)$. The CQM does not predict the existence of such a zone for the step potential, thereby leading to the discontinuous average velocity of transmitted particles.

The SWF ψ_{ref} to obey the requirements (a)-(c) reads as follows. For $x < a$

$$\psi_{ref}(x,k) = A_{ref}e^{ikx} + Be^{-ikx} \tag{2}$$

where $A_{ref} = \sqrt{R}e^{i\beta\lambda}$; $\lambda = \arctan\sqrt{T/R}$. For $a < x < x_c$

$$\psi_{ref}(x,k) = -2\beta^{3/2}\sqrt{\frac{k}{\kappa}R}\ \sin[\kappa(x - x_c)]e^{i(\kappa a + \beta\lambda/2)}. \tag{3}$$

For $x > x_c$, $\psi_{ref} \equiv 0$. As regards $\psi_{tr}(x,k)$, this SWF can be calculated from Eqs. (1)–(3) due to the property (a): $\psi_{tr}(x,k) = \Psi_{tot}(x,k) - \psi_{ref}(x,k)$.

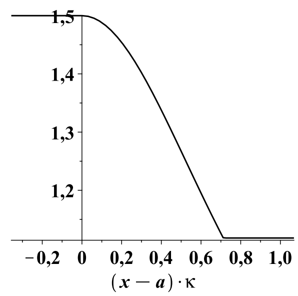

Figure 1. The relative transmission velocity $v_{tr-flow}/v_0$ for $V_0 > 0$ and $k/\kappa_0 = 1.5$; $v_0 = \hbar\kappa_0/m$

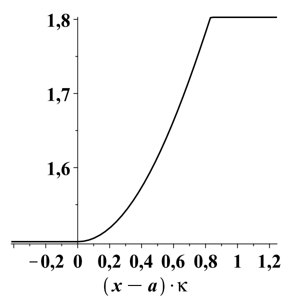

Figure 2. The relative transmission velocity $v_{tr-flow}/v_0$ for $V_0 < 0$ and $k/\kappa_0 = 1.5$

3. Characteristic times for transmission and reflection

The study of the temporal aspects of both subprocesses is based here on the flow and group velocity concepts. We begin with the former because, by our approach, only this concept allows us to reveal the average velocity of particles passing above the step, as well as to define the corresponding delay time.

3.1. Dwell times for transmission and reflection

Since the transmission dynamics is known in all spatial regions, we can define now the transmission velocity at any point on the OX axis: $v_{tr-flow}(x,k) = I_{tr}/|\psi_{tr}(x,k)|^2$; $I_{tr} = I_{tot} = \hbar k T(k)/m$. Figs. 1 and 2 obtained for the interval $[a - x_c/2, x_c + x_c/2]$ show that in the transient zone $[a, x_c]$ the flow velocity of transmitted particles varies monotonously between the 'free-particle' velocities $v_k = \hbar k/m$ and $v_\kappa = \hbar\kappa/m$. When $V_0 > 0$ ($V_0 < 0$), the average velocity of transmitted particles in this region is larger (smaller) than v_κ.

Simple calculations show that the transmission dwell time $\tau_{tr-dwell} = \int_a^{x_c} |\psi_{tr}(x,k)|^2 dx/I_{tr}$ to characterize the dynamics of transmitted particles in the transient zone $[a, x_c]$ reads as

$$\tau_{tr-dwell} = \frac{m}{2\hbar k\kappa^3}\left[(\kappa^2 + k^2)\arctan\sqrt{\frac{\kappa}{k}} + (\kappa - k)\sqrt{k\kappa}\right]$$

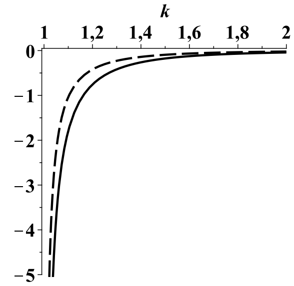

Figure 3. The delay times $\tau_{tr-group-del}$ (solid line) and $\tau_{tr-dwell-del}$ (dashed line) for $V_0 > 0$, in units of τ_0 where $\tau_0 = m/\hbar\kappa_0^2$

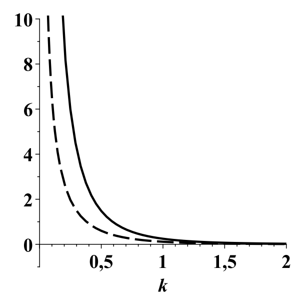

Figure 4. The same quantities as in fig. 3, but for $V_0 < 0$

Thus, the dwell delay time $\tau_{tr-dwell-del} = \tau_{tr-dwell} - \tau_{free}$, where $\tau_{free} = (x_c - a)/v_\kappa$, is

$$\tau_{tr-dwell-del} = \frac{m(\kappa - k)}{2\hbar k\kappa^3}\left[(\kappa - k)\arctan\sqrt{\frac{\kappa}{k}} + \sqrt{k\kappa}\right] \quad (4)$$

This quantity is negative for $V_0 > 0$ and positive for $V_0 < 0$ (see dashed curves in figs. 3 and 4, respectively).

Of course, the above flow-velocity concept cannot be applied exactly to reflection, because $I_{ref} = 0$. This zero probability density is usually substituted by that of the corresponding incident flow (see, e.g., ([13])). In line with this rule, we define the reflection dwell time for the transient zone as follows,

$$\tau_{ref-dwell} = \frac{1}{I_{ref}^{inc}}\int_a^{x_c} |\psi_{ref}(x,k)|^2 dx.$$

where $I_{ref}^{inc} = R(k)\hbar k/m$. Considering Eq. (3) we obtain

$$\tau_{ref-dwell} = \frac{2m}{\hbar\kappa^2}\left[\arctan\sqrt{\frac{\kappa}{k}} - \frac{\sqrt{\kappa k}}{k+\kappa}\right]. \quad (5)$$

Then, assuming that reflected particles move in the region $x > a$ with the velocity v_κ, and $\tau_{ref-dwell} = 2l_{depth}/v_\kappa$, we obtain from Eq. (5) that

$$l_{depth} = \frac{1}{\kappa}\left[\arctan\sqrt{\frac{\kappa}{k}} - \frac{\sqrt{\kappa k}}{k+\kappa}\right]. \quad (6)$$

Note that the *average* turning point for reflected particles lies always to the left of their *extreme right* turning point: $a + l_{depth}(k) \le x_c(k)$. For $V_0 > 0$, the ratio $l_{depth}/(x_c - a)$

tends to zero when $k \to \kappa_0 + 0$; but it tends to $1 - 2/\pi$ from below, when $k \to \infty$. For $V_0 < 0$, the ratio $l_{depth}/(x_c - a)$ tends to unit when $k \to 0$; but it tends to $1 - 2/\pi$ from above, when $k \to \infty$.

So, for a particle scattering on the potential step the concept of the flow velocity leads, within our approach, to nonzero transmission and reflection delay times. For transmitted particles this effect takes place due to the transient zone where their average velocity smoothly varies between the values v_k and v_κ.

3.2. Group times for transmission and reflection

Our next aim is to study the temporal aspects of both subprocesses in the time-dependent case. For this purpose the group-velocity concept will be used.

Let the time-dependent wave function $\Psi_{tot}(x,t)$ to describe the whole scattering process be

$$\Psi_{tot} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\Psi_{tot}(x,k)e^{-iE(k)t/\hbar}dk$$

where $\mathcal{A}(k)$ is, for example, the Gaussian function: $\mathcal{A}(k) = (2l_0^2/\pi)^{1/4}\exp(-l_0^2(k-k_0)^2)$. In this setting, the wave function represents at the initial time $t = 0$ the wave packet of width l_0 ($l_0 \ll a$), with the CM positioned at the point $x = 0$.

The incident $\Psi_{inc}(x,t)$, transmitted $\Psi_{tr}(x,t)$ and reflected $\Psi_{tr}(x,t)$ wave packets are

$$\Psi_{inc} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\, e^{i[kx - iE(k)t]/\hbar}dk,$$

$$\Psi_{tr} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\sqrt{\frac{k}{\kappa}T(k)}\, e^{[i(\kappa x + (k-\kappa)a - E(k)t/\hbar)]}dk,$$

$$\Psi_{ref} = \frac{\beta}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\sqrt{\frac{k}{\kappa}R(k)}\, e^{[i(k(2ka - x) - E(k)t/\hbar)]}dk.$$

The corresponding time-dependent SWFs $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$ read as

$$\psi_{tr,ref} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\psi_{tr,ref}(x;k)\, e^{-iE(k)t/\hbar}dk.$$

These time-dependent SWFs are evident to satisfy the requirement (a).

The incident wave packets ψ_{tr}^{inc} and ψ_{ref}^{inc} for transmission and reflection differ from Ψ_{inc}. They are, respectively,

$$\psi_{tr}^{inc} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\sqrt{T(k)}\, e^{\left[i\left(kx + \beta\left(\lambda(k) - \frac{\pi}{2}\right) - E(k)t/\hbar\right)\right]}dk$$

$$\psi_{ref}^{inc} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\sqrt{R(k)}\, e^{[i(kx + \beta\lambda(k) - E(k)t/\hbar)]}dk$$

As regards the scattered wave packets ψ_{tr}^{fin} and ψ_{ref}^{fin} for transmission and reflection, $\psi_{tr}^{fin}(x,t) \equiv \Psi_{tr}(x,t)$ and $\psi_{ref}^{fin}(x,t) \equiv \Psi_{ref}(x,t)$. This means that the time of arrival of transmitted (reflected) particles at some distant point x in the transmission (reflection) region is the same both in our model and in the CQM.

For each subprocess the incident wave packet approximates the corresponding wave function at the initial stage of scattering, and the scattered one approaches it at the final stage; in both cases $|x-a| \gg l_0$. Thus, in terms of these wave packets we can explicitly define the *asymptotic* scattering times for each subprocess.

Let us do this for the asymptotically large interval $[0, a+L]$ where $L \gg l_0$. Standard calculations, with making use of the equality $\delta[\kappa(k') - \kappa(k)] = \delta(k'-k)\kappa(k)/k$, yield

$$< \psi_{tr}^{inc}|\psi_{tr}^{inc} >=< \Psi_{tr}|\Psi_{tr} >= \int_{-\infty}^{\infty} |\mathcal{A}(k)|^2 T(k) dk \equiv \mathcal{T}$$
$$< \psi_{ref}^{inc}|\psi_{ref}^{inc} >=< \Psi_{ref}|\Psi_{ref} >= \int_{-\infty}^{\infty} |\mathcal{A}(k)|^2 R(k) dk \equiv \mathcal{R}.$$

That is, the norm of each subprocess wave function has the same constant value at the initial and final stages of scattering.

We have to stress once more (see [8, 9, 11]) that, in the strict limit $l_0 \to \infty$, $\mathcal{T} = T(k_0)$ and $\mathcal{R} = R(k_0)$ for all stages of scattering. Thus, in this case $\mathcal{T} + \mathcal{R} = 1$. But for a however large but finite value of l_0 this equality is violated at the stages when the front or tail parts of the wave packet $\psi_{tr}(x,t)$ crosses the joining point $x_c(k_0)$. At these stages the norm $\mathcal{T}$ varies. What influences the velocity of the CM of the wave packet $\psi_{tr}(x,t)$. In more detail this feature of the transmission subprocess has been discussed in [8].

Let us now present the CM's dynamics of both wave packets at the initial and final stages of scattering, when l_0 is large enough:

$$< x >_{tot}^{inc}= \frac{\hbar k}{m} t, \quad < x >_{tr}^{inc}=< x >_{ref}^{inc}= \frac{\hbar k}{m} t - \beta \frac{d\lambda}{dk},$$
$$< x >_{tr}^{fin}= \frac{\hbar \kappa}{m} t + a\left(1 - \frac{\kappa}{k}\right), \quad < x >_{ref}^{fin}= -\frac{\hbar k}{m} t + 2a \tag{7}$$

here k_0 is substituted by k; $< x >_{\psi}=< \psi|\hat{x}|\psi > / < \psi|\psi >$; $\hat{x}$ is the operator of the particle position.

Let t_{dep}^{tr} and t_{dep}^{ref} be, respectively, the times of the departure of transmitted and reflected particles from the point $x=0$. Let also t_{ar}^{tr} and t_{ar}^{ref} be their times of arrival at the points $x=L$ and $x=0$, respectively. As is seen from Eqs. (7), the CMs of the (narrow in k space) transmitted and reflected wave packets start from the same point

$$< x >_{tr}^{inc}(0) =< x >_{ref}^{inc}(0) = x_{start} = -\beta \cdot d\lambda/dk, \tag{8}$$

which does not coincide with the initial position $x=0$ of the CM of the total wave packet $\Psi_{tot}(x,t)$, as is assumed in the CQM. Thus, the CMs of the wave packets ψ_{tr} and ψ_{ref} should start from the point $x=0$ at the same nonzero instant of time $t_{dep}^{tr} = t_{dep}^{ref} = t_{dep}$ which obeys the equation

$$\frac{\hbar k}{m} t_{dep} - \beta \frac{d\lambda}{dk} = 0. \tag{9}$$

As regards the arrival times t_{ar}^{tr} and t_{ar}^{ref}, they are individual for each subprocess:

$$-\frac{\hbar k}{m} t_{ar}^{ref} + 2a = 0, \quad \frac{\hbar \kappa}{m} t_{ar}^{tr} + a\left(1 - \frac{\kappa}{k}\right) = a + L. \tag{10}$$

Thus, $\Delta t_{tr} = t_{ar}^{tr} - t_{dep}$ and $\Delta t_{ref} = t_{ar}^{ref} - t_{dep}$ should be treated, respectively, as the group transmission and reflection times for the interval $[0, L]$. Considering Eqs. (9) and (10), we obtain

$$\Delta t_{tr} = \frac{mL}{\hbar\kappa} + \frac{ma}{\hbar k} - \beta\frac{m}{\hbar\kappa}\frac{d\lambda}{dk}, \quad \Delta t_{ref} = \frac{m}{\hbar k}\left(2a - \beta\frac{d\lambda}{dk}\right).$$

Then, excluding the terms to contain L and a, we obtain the *asymptotic* group delay times $\tau_{tr-group-del}$ and $\tau_{ref-group-del}$ for transmission and reflection, respectively. They are equal:

$$\tau_{tr-group-del} = \tau_{ref-group-del} = \frac{m(\kappa - k)}{\hbar(k\kappa)^{3/2}}; \tag{11}$$

note that $k > \kappa$ for $V_0 > 0$, and $k < \kappa$ for $V_0 < 0$.

As is seen in figs. 3 and 4, the *asymptotic* group delay time $\tau_{tr-group-del}$ and the dwell delay time $\tau_{tr-dwell-del}$ have the same signs. However, $|\tau_{tr-group-del}| > |\tau_{tr-dwell-del}|$. In the limit $k \to \infty$ the ratio $\tau_{tr-group-del}/\tau_{tr-dwell-del}$ tends to 2, irrespective of the step's sign. While in the opposite limit its sign is important: for $V_0 > 0$ the ratio tends to 3/2 when $k \to \kappa_0 + 0$, but for $V_0 < 0$ the ratio diverges as $k^{-1/2}$ when $k \to 0$.

Of course, it should be stressed that the analysis of the ratio $\tau_{tr-group-del}/\tau_{tr-dwell-del}$ in the limit $k \to 0$ is formal, because the group delay $\tau_{tr-group-del}$ describes the wave packets taking part in a *completed* scattering. This implies that when we deal with the wave-packet the velocity v_k must be large enough, otherwise the transmitted and reflected wave packets will overlap each other in the limit $t \to \infty$ (this also means that the condition $x_{start}(k) < a$ should always be performed).

The fact that $\tau_{tr-group-del}$ and $\tau_{tr-dwell-del}$ do not coincide with each other is not surprising because these quantities have the different physical meanings: the former characterizes the influence of the potential step on the *CM* of the transmitted wave packet during the whole time-dependent scattering process; while the latter describes the quantum dynamics of transmitted *particles* in the region $[a, x_c]$ to lie in the neighborhood of the step. Of importance is to stress once more – because of the nonunitary character of the transmission dynamics at the very stage of scattering, the CM velocity of the transmitted wave packet cannot serve as a characteristic time of transmitted particles.

3.3. Scattering times for a complete reflection

Let us now consider the case $V_0 > 0$ when a particle has a well-defined energy E to belong to the interval $(0, V_0)$, i.e., when $T(k) = 0$. Now, in Eqs. (1)

$$A = \frac{2k}{k + i\kappa}e^{(\kappa + ik)a}, \quad B = \frac{k - i\kappa}{k + i\kappa}e^{2ika}$$

where $\kappa = \sqrt{\kappa_0^2 - k^2}$. Simple calculations yield that now

$$\tau_{ref-dwell} = \frac{2mk}{\hbar\kappa\kappa_0^2}, \quad \tau_{ref-group-del} = \frac{2m}{\hbar k\kappa}. \tag{12}$$

When $E \to V_0 - 0$ (i.e., when $\kappa \to 0$), $\tau_{ref-dwell} \approx \tau_{ref-group-del} \approx 2m/\hbar\kappa_0\kappa$.

Note, in the case considered there is no splitting of the incident wave packet into two parts. Therefore the group delay time $\tau_{ref-group-del}$ (12) coincides with the term $2d/\nu$ in Exp.(4) of the paper [2].

However, in the limit $E \to V_0 + 0$ ($V_0 > 0$), when the mentioned splitting exists, our approach leads to different results. By [1, 2] the transmission and reflection *group* delay times are zero in this limit, while our approach predicts nonzero values:

$$\frac{2}{3}\,\tau_{tr-dwell-del} \approx \tau_{tr-group-del} \approx -\frac{2m\kappa_0}{\hbar(\kappa_0\kappa)^{3/2}}$$

As regards $\tau_{ref-dwell}$, within the CQM this time scale has not been considered for the potential step.

4. Discussion and conclusion

In this paper we present a novel model of scattering a quantum particle on the potential step, which allows tracing the dynamics of its subprocess – transmission and reflection – at all stages of scattering. On the basis of this model we define characteristic times for both subprocess – dwell times and asymptotic group times. By the conventional model, the average velocity of a particle to travel above the step is a piecewise constant function of x, what is unacceptable for particles with the nonzero rest mass. By our approach for a particle with a well-defined energy $E(k)$ there is a transient zone immediately behind the step, where the average particle velocity varies monotonously from v_k to v_κ. Due to this zone the delay time calculated for transmitted and reflected particles is nonzero for the potential step, while they must be zero by the conventional model.

Of course, the flow and group velocity concepts do not give a complete picture of the subprocess dynamics for this potential. Of importance is also to apply to this problem the Larmor timekeeping procedure, which is assumed to allow one to (directly or indirectly) measure the transmission and reflection dwell times.

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